Family labor supply, commuting time, and residential decisions: the case of the Tokyo Metropolitan Area *

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Abstract

In this paper, I build a model of family labor supply and residential choices that explicitly incorporates the full-time or part-time work decisions of married women. The model can explain why women’s participation patterns in full-time and part-time work vary significantly in areas that are geographically close but differ in real estate prices. The model suggests that high commuting costs could be one of the main obstacles for women’s full-time employment in places like the Tokyo Metropolitan Area.

JEL classification: J22, R21, R23
Key words: Commuting, family labor supply, part-time work

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1 Introduction

How is women’s employment related to the geographical distribution of households? The present paper explores this question in a specific context: the Tokyo Metropolitan Area in Japan. I present an optimizing model of family labor supply and residential choices that incorporates an important institutional feature of the Japanese labor market: the decision by women to work full time or part time. The Tokyo Metropolitan Area is also unique in that real estate prices are much higher for locations close to the Central Business District (CBD).

Costa and Kahn (2000) show that highly educated “power couples” in the United States became more likely to locate in the Metropolitan Statistical Areas (MSAs) from 1940 to 1990. They attribute this change to the colocation problem, i.e., the difficulty of finding a residence convenient to both spouses’ workplaces. In contrast, Compton and Pollak (2007) analyze panel data and show that power couples are no more likely than other types to migrate to large metropolitan areas. Accordingly, they argue that colocation is not a major cause of the concentration of power couples in large MSAs.¹ Neither of these studies, however, is very explicit about the relationship between residential decisions and housing prices.

Several studies have investigated the relationship between women’s employment and housing purchases. Yoshikawa and Ohtake (1989), Fortin

¹Compton and Pollak (2007) also show that the concentration of power couples in MSAs declined slightly between 1990 and 2000.
(1995), and Del Boca and Lusardi (2003) empirically examine how the working status of married women is related to the housing purchases of their households. In the regional science literature, Mok (2007) asks whether incomes of spouses are pooled or not in deciding housing location. Using the Canadian Census data for the Toronto Metropolitan Area, she finds that incomes are not pooled for couples without children. Iwata and Tamada (2008) build a model of commuting behavior of married women and argue for a non-monotonic relationship between the wage rate of working married women and their commuting time. Sakanishi (2007) builds a model in which household location is determined by the husband’s choice and the wife chooses her employment status given the husband’s location.

In this paper I present a model of family labor supply that permits married women to choose from full-time and part-time job opportunities. The commuting costs, wage levels, and working hours for the two employment options are explicitly modeled so as to reflect the real constraints faced by couples in Japan. The model explains why the distribution of women’s employment status differs significantly across prefectures in the Tokyo Metropolitan Area.

The paper makes two novel contributions to the literature. First, it explicitly considers the optimization over married women’s work (either full-time or part-time) and residential location. In modeling this, the “discreteness” of these two choices is particularly emphasized. This element of the model is especially relevant in the Tokyo Metropolitan Area, where full-time work is
more common among women living close to the CBD than among women living in the suburbs. Such regional differences in participation are not observed for men (see Section 2). By modeling the family’s housing location and labor supply decisions simultaneously, this model provides a natural explanation for such patterns.

Second, the paper highlights the importance of commuting costs in understanding women’s work in Japan. Several important measures (legal and otherwise) to promote women’s participation in the workplace have been adopted in Japan in recent decades (e.g., the Equal Employment Opportunity Law, the Maternity Leave Law, and various policies to make balancing work and family easier). Nonetheless, the proportion of women working in regular full-time jobs in Japan has not increased much, especially after age 40. Most of the increase in women’s employment after middle age has taken the form of non-regular, part-time work (Abe 2009b). Commuting costs (including the time and energy necessary for commuting) have not fallen much for female workers in the past several decades, which may explain the sluggish advancement in women’s regular full-time employment.

The unique aspect of the model is that it explicitly incorporates the discrete feature of the labor market. In Japan, full-time wages are much higher than part-time wages. Furthermore, it is difficult for women who left the labor force at childbearing age to return to regular employment (Ueda 2007). These factors make full-time and part-time work very distinct, whereas the middle ground between these two options is almost nonexistent. The model
shows that the housing market operates in a way to further strengthen this
discrete feature in the labor market.

Discreteness in the labor market can sometimes be mitigated by other
“markets.” For instance, if the full-time wage is higher than the part-time
wage, in a multi-period setting full-time workers retire early but part-time
workers retire late. Then lifetime earnings from the two employment statuses
are more equalized than for single period earnings, implying that intertemporal
substitution mitigates discreteness in a single period.\footnote{Abe (2009a), albeit in a
different setting from this paper, presents a model that explains this point.}
What is unique about the housing market in this paper is that it operates to magnify the
discreteness in the labor market.

The property that high wage workers live close to the CBD while low wage
workers live farther from the CBD is common in many monocentric models,
because the bid-rent gradient is higher for high wage earners. In contrast, the
model of this paper assumes that all men work at CBD while women have
options to engage in part-time employment, for which the commuting time is
zero. In this model, the distance from the CBD creates larger differences on
women’s behavior than for men’s. The large discrepancy in women’s behavior
is caused by the discreteness in the labor market, magnified by the housing
market.

This paper is organized as follows. Section 2 presents stylized facts on
regional differences in participation in the labor market. Section 3 presents
a model that explains the stylized facts presented in Section 2. Section 4 provides a numerical example of the model. Section 5 concludes.

2 Stylized Facts on regional differences of labor force behavior

2.1 Participation in full-time and part-time work by region

Figures 1.A and 1.B show participation patterns in the four prefectures in the Tokyo Metropolitan Area: Saitama, Chiba, Tokyo, and Kanagawa. Tokyo prefecture includes the Tokyo CBD; the other three prefectures are generally suburban, although they do contain some large cities such as Saitama (in Saitama prefecture), Yokohama and Kawasaki (in Kanagawa prefecture).

In the following discussion, three measures of participation are used: the employment-to-population ratio (E-P ratio), the regular employment ratio, and the part-time employment ratio. The regular employment ratio is the number of regular employees divided by population, for which regular employees include “regular” wage and salary earners, as well as executives of private corporations. The part-time employment ratio is the number of part-time and casual employees divided by population. Part-time and casual

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3In the data used in this subsection, “part-time” and “casual” are the terms for employment status in the workplace. Both part-time and casual employees are non-regular workers. There is no clear distinction in tasks or working conditions for the two types
employees comprise the typical non-regular workforce in Japan and their numbers have increased dramatically in recent decades. Regular employees are those with no term limits in their contracts. Part-time and casual employees typically hold a contract of no longer than 1 year in duration, although such contracts are often renewed multiple times. Regular employees are eligible for employer-provided fringe benefits, including public pension contributions, a corporate pension, and employer-provided health insurance; part-time and casual employees are much less likely to obtain such coverage. Regular employees normally work 40 hours per week or more, whereas part-time and casual employees work fewer hours. Hourly wage rates are higher for regular employees than for part-time employees.4

Figures 1.A and 1.B plot the three measures against age for women and men, using aggregate data from the ESS in 2007.5 The data show several interesting patterns. First, the E-P ratio for women below age 40 is high in Tokyo than in the three other suburban prefectures. This ratio is similar in all four prefectures after age 40. The types of women's employment observed in the four regions are quite different: regular employment is more common in Tokyo, whereas part-time employment is more common in the other three

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\[\text{of workers, although casual workers are generally younger than part-time workers. Students who work part time normally work under casual status. In the questionnaire of the Employment Status Survey (ESS, Statistics Bureau, Ministry of Internal Affairs and Communications of Japan), casual workers are referred as “arbeit workers.”}\]

\[\text{An extensive literature on part-time jobs in Japan provides evidence for these issues; see, e.g., Houseman and Osawa (1998, 2003), Abe (2003), Gaston and Kishi (2007).}\]

\[\text{I omit the part-time employment ratio for men since this ratio is generally low for men.}\]
prefectures, independent of age. Second, the E-P ratio and the regular employment ratio for men do not differ much across the four prefectures. If anything, the regular employment ratio for men is slightly lower in Tokyo than elsewhere. Overall, the regional differences in women’s employment are clearly larger. It is natural to expect that these significant differences are caused by endogenous choice of residential location. Housing prices are higher in Tokyo and full-time employees earn much more than part-time employees. On the other hand, full-time jobs carry less flexible work schedules. High earnings from full-time working wives may be used to finance the high cost of housing close to the CBD. Households in which the wife does not work full time will locate farther from the CBD to spend less for housing.

2.2 Educational composition and labor supply choices across prefectures

Most large-scale datasets for Japanese workers do not contain information on both wage and place of residence, so the hypothesis laid out in the previous section is difficult to be tested directly. In this section, I therefore use education as a proxy for wage levels. It is well known that education is highly correlated with full-time wages. I first look at the proportion of educated people among residents, and then consider the proportion of educated

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6 More generally, it is known that women’s participation in the labor market, especially with respect to regular full-time work, has significant regional variations within Japan (Takeishi 2007; Abe et al. 2008). Regional variations are small for men.
workers among residents.\textsuperscript{7}

Figure 2 shows the proportion of people with more than a university education (16 years or more) among residents in the four prefectures by age group. Among both men and women of all age groups, the proportion of highly educated persons ("power" individuals in the terminology of Costa and Kahn (2000) and Compton and Pollak (2007)) is highest in Tokyo, followed by Kanagawa, Chiba, and Saitama. These data support the idea that high-wage earners tend to live closer to the Tokyo CBD.

The educational distribution does not show whether labor supply is related to residential choice because the population considered includes both workers and non-workers.\textsuperscript{8} To see the distribution of "power workers," the proportion of those with more than a university degree and working (power working men and women) among residents is plotted in Figure 3. For both men and women, the proportion of power workers is highest in Tokyo. The pattern in Figure 3 basically mirrors the educational distribution (Figure 2), so it can be concluded that labor supply choices do not have a large impact on the geographical distribution of power working men and women; the effect of education is more significant.

\textsuperscript{7}Costa and Kahn (2000) define "power couples" as couples in which both spouses have more than a college degree. The available data do not allow me to investigate the combination of educational attainment of husband and wife. In this paper, "power" men and women are those who have at least a university degree, and "power working" men and women are those who have at least a university degree \textit{and} work.

\textsuperscript{8}In Japan, the labor force participation rate of highly educated women is not much higher than that of less educated women.
2.3 Housing prices, educational composition, and labor supply

More evidence on educational attainment and labor supply choices by residents is provided by aggregate data from the 2000 Census. Census data are available at the municipal level; here, I focus on cities in the Tokyo prefecture. The Census data on educational attainment and labor supply can be matched with housing price data.

I estimate regressions relating the proportion of power working men and women to housing prices. The dependent variable is the proportion of power working men or women divided by the population. In defining the number of power working men and women, two types of workers are considered: one includes all types of employment, and the other is restricted to wage and salary earners. The independent variables are the log of mean land price and age dummies. Data for the mean land prices (per m\(^2\)) at the municipal level are obtained from the Public Notice of Land Prices (Ministry of Land, Infrastructure, Transport and Tourism of Japan) in 2000. Results are shown in Table 1. Land prices have a positive impact on the proportion of power working men and women: a 10 percent increase in price is associated with a 1-2 percent increase in the dependent variable. The coefficients are larger for men than for women.\(^{10}\)

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\(^{9}\)The aggregate municipal data are available only for municipalities with a population of more than 0.5 million. There are 8 such municipalities in Tokyo prefecture.

\(^{10}\)Causality should not be inferred, as this pattern of the distribution of power working men and women is consistent with several other explanations. For example, highly edu-
I also run regressions in which the dependent variable is the labor force participation rate and the independent variables are the log housing price and age dummies. The results are shown in Table 2. Among men and highly educated women, housing prices do not affect participation. Therefore, labor supply choices of these people do not differ across regions, which is consistent with the observation that the distributions of power working men and women is similar to the distributions of power men and women. Among women with a senior high school degree, those who live in areas with high housing prices are more likely to participate in the labor market.

3 Model

3.1 Basic model

Consider a household with two adult members, a husband and a wife. The family utility is defined as $U(l_h, l_w, C)$, where $l_h$ is the husband’s leisure time, $l_w$ is the wife’s leisure time, and $C$ is the value of consumption other than housing. In the rest of this paper, the first derivative of $U(.)$ with respect to spouse $j$’s leisure is denoted as $U_j$ $(j = h, w)$, and the first derivative with respect to $C$ is denoted as $U_C$. The second derivatives are denoted similarly ($U_{Cw}, U_{wh}$, etc.). Let $W_h$ represent the husband’s hourly wage rate, and let $E_h$ represent his earnings. The model has a monocentric structure cated people prefer to live in areas where school quality is high, and school quality and housing prices are positively correlated.
with a single CBD. It is assumed that the husband works in the CBD and his commuting distance $D$ is measured in hours. Let $\bar{D}$ be the maximum possible value of $D$: thus, $D \in [0, \bar{D}]$.

The wife’s employment choice is either full-time work, part-time work, or no work. Full-time work requires the wife to work more than $\theta$ hours. All full-time jobs are located in the CBD, so the commuting distance for any full-time job is $D$. The wife’s hourly wage rate when she works full time is $W_F$. Part-time jobs are available at zero commuting distance, for which the wife’s hourly wage rate is $W_P$. It is assumed that hours of work are chosen as a continuous variable, so the earnings function for the wife (the function that relates her working hours, $h_w$, to her earnings, $E_w$) can be modeled as:

$$E_w(h_w) = \begin{cases} W_P h_w & \text{if } h_w \leq \theta \\ W_F h_w & \text{if } h_w > \theta \end{cases}$$

(1)

Note that there is a “jump” in the earnings function at point $\theta$. This is natural in the context of the Japanese labor market, where the hourly wage rate of female full-time employees is approximately 30 percent higher than that of female part-time employees.

The time endowment for both husband and wife is $\bar{L}$ and their time constraints give

$$l_h = \bar{L} - h_k - D.$$
\[ l_w = \bar{L} - h_w - I(h_w > \theta) \cdot D, \]

where \( h_j \) is the working hours of spouse \( j \), \( D \) is commuting time to the CBD, and \( I(.) \) is an indicator function that takes value 1 if the relation inside the parentheses is true and zero otherwise. For simplicity, I assume that all housing units are of the same size and the incomes of the husband and wife are assumed to be pooled.

There are four possible solutions to the optimization problem described above: (1) the wife does not work \((h_w = 0)\); (2) the wife works part time with fewer than \( \theta \) hours \((0 < h_w < \theta)\); (3) the wife works exactly \( \theta \) hours as a part-time worker \((h_w = \theta)\); or (4) the wife works full time \((h_w > \theta)\). When necessary, I use the following subscripts to denote the four states: \( N \) for no-work, \( I \) for “interior” part-time work fewer than \( \theta \) hours \((0 < h_w < \theta)\), \( T \) for part-time work at the threshold, and \( F \) for full-time work. Among these four, the first three \((N, I, \text{ and } T)\) are collectively designated as “part-time work,” because “no work” and “threshold” are at the boundaries of the budget set and form corner solutions in the maximization problem for part-time work.

To derive the optimal location and labor supply choice, two sub-problems (the part-time and full-time sub-problems) are defined, and the bid-rent function for housing is derived for each.\(^{11}\) The bid-rent function of the part-time sub-problem is derived by solving the following problem:

\[ \begin{align*}
\max_{h_h, h_w, C} & \quad w_h h_h + W_P h_w - C \\
\end{align*} \quad (2) \]

\(^{11}\)The derivation in this section follows that in Fujita (1989).
subject to
\[ U(\bar{L} - h_h - D, \bar{L} - h_w, C) = u, \]
\[ 0 \leq h_w \leq \theta, \]
where \( u \) is the utility level of the household. The maximand of this problem gives the bid-rent function for the part-time sub-problem, denoted \( \psi^P(D, u) \). Likewise, the bid-rent function for the full-time sub-problem is derived by solving the following:

\[
\text{Max}_{h_h, h_w, C} \quad w_h h_h + W_F h_w - C \tag{3}
\]
subject to
\[ U(\bar{L} - h_h - D, \bar{L} - h_w - D, C) = u, \]
\[ h_w \geq \theta. \]

The maximand of this problem gives the bid-rent function for the full-time sub-problem, denoted \( \psi^F(D, u) \).\(^{12}\)

Let \( \tilde{C}(l_h, l_w, u) \) represent the consumption level that achieves utility level \( u \) when the husband’s leisure is \( l_h \) and the wife’s leisure is \( l_w \). The first-order condition with respect to \( h_h \) is the same for the two sub-problems, and is

\(^{12}\)In the entire problem, the choice of \( h_w = \theta \) is included in the part-time status and not in the full-time status. However, to make the full-time sub-problem well-defined (i.e., to make the opportunity set a closed set), \( h_w = \theta \) is included in the opportunity set for the full-time sub-problem.
expressed as:
\[ W_h + \frac{\partial \tilde{C}(.)}{\partial h} = 0 \] (4)

The first-order condition with respect to \( h_w \) in the full-time sub-problem is:
\[ W_F + \frac{\partial \tilde{C}(.)}{\partial l_w} \leq 0, \] (5)
for which the inequality holds when \( h_w = \theta \).

The first-order condition with respect to \( h_w \) in the part-time sub-problem is as follows:
\[
\begin{cases}
W_P + \frac{\partial \tilde{C}(l_h, \bar{T}, u)}{\partial l_w} < 0, & \text{if } h_w = 0 \quad [N] \\
W_P + \frac{\partial \tilde{C}(l_h, h_w, u)}{\partial l_w} = 0, & \text{if } 0 < h_w < \theta \quad [I] \\
W_P + \frac{\partial \tilde{C}(l_h, \bar{T} - \theta, u)}{\partial l_w} > 0, & \text{if } h_w = \theta \quad [T]
\end{cases}
\] (6)

where the letter in square brackets indicates one of the four states.

The following two conditions are met at the corner solution of \( h_w = \theta \):
\[ W_P + \frac{\partial \tilde{C}(l_h, \bar{T} - \theta, u)}{\partial l_w} > 0, \]
\[ W_F + \frac{\partial \tilde{C}(l_h, \bar{T} - \theta - D, u)}{\partial l_w} < 0. \]

The “discreteness” in the labor market is represented by this corner solution, which is the unique feature of this model. A wife’s full-time work requires \( D \) hours of commuting time, in addition to the hours spent at her
job. Suppose the wife changes her employment status from part time to full time and increases her working hours by $\Delta h_w$ (from less than $\theta$ to over $\theta$). Then her leisure time decrease by $D + \Delta h_w$. $D$ is essentially the “fixed cost of full-time work,” measured in hours. Unless $D = 0$, there will be a range of $W_P$ and $W_F$ for which the wife’s working hours are set to $\theta$ (part-time work) and do not increase even if $W_P$ rises (the wage elasticity of working hours is zero). At this corner solution, $-\partial \tilde{C}(l_h, L - \theta, u) / \partial l_w < W_P < W_F \leq -\partial \tilde{C}(l_h, L - \theta - D, u) / \partial l_w$ holds. The wife works as many hours as possible while remaining part-time; she would like to work more than $\theta$ hours but commuting prevents her from pursuing full-time employment opportunities.

The derivative of the bid-rent function is the time cost of commuting, implying

$$\frac{\partial \psi^P}{\partial D} = -W_h, \quad \text{if } h_w \leq \theta$$

$$\frac{\partial \psi^F}{\partial D} = -(W_h + W_F), \quad \text{otherwise}$$

This is the familiar Alonso-Mills-Muth (AMM) condition.\(^{13}\) The transportation cost per hour is the opportunity cost of working and is equal to the hourly wage rate of full-time workers who commute to the CBD.

**Assumption 1.** Let $\Delta C(l_h, u) = \tilde{C}(l_h, L - \theta - D, u) - \tilde{C}(l_h, L - \theta, u)$, where $l_h > 0$. Then, for some households, $(W_F - W_P)\theta < \Delta C(l_h, u)$ for all $l_h > 0$ and $u$.

To understand the implication of Assumption 1, suppose a household is

\(^{13}\)Alonso (1964), Mills (1967), and Muth (1969).
located at $\bar{D}$. $\Delta C(.)$ is the compensation necessary for the wife to work $\theta$ hours as a full-time worker (commute $\bar{D}$ hours), and to keep the household equally well-off when she has part-time status and work $\theta$ hours. Assumption 1 says that for some households, the part-time/full-time wage gap is not large enough to compensate for the wife’s commute of $\bar{D}$ hours for any possible values of $l_h$ and $u$.

**Proposition 1. For all households,**

$$\psi^P(0, u) < \psi^F(0, u).$$

*For households for which Assumption 1 holds,*

$$\psi^P(\bar{D}, u) > \psi^F(\bar{D}, u).$$

*The bid-rent function for a household is*

$$\psi(D, u) = \max(\psi^P(D, u), \psi^F(D, u)).$$

*Proof. When $D = 0$, the maximization problem for deriving the bid-rent functions for part-time and full-time status differ only in the level of the wife’s wage (whether it is $W_P$ or $W_F$). Since $W_F > W_P$, it is obvious that $\psi^P(0, u) < \psi^F(0, u)$.*

The bid-rent functions at location $\bar{D}$, for the full-time and part-time cases
are as follows:

$$\psi^F(\mathcal{D}, u) = W_h(\mathcal{L} - t^F_h - \mathcal{D}) + W_F h^F_w - \tilde{C}(t^F_h, \mathcal{L} - h^F_w - \mathcal{D}, u),$$  \hspace{1cm} (9)$$

$$\psi^P(\mathcal{D}, u) = W_h(\mathcal{L} - t^P_h - \mathcal{D}) + W_P h^P_w - \tilde{C}(t^P_h, \mathcal{L} - h^P_w, u),$$  \hspace{1cm} (10)$$

where $t^k_h (k = F, P)$ represents the husband’s optimal leisure in the $k$-th sub-problem. Since $h^F_w$ is chosen optimally, $\psi^P(\mathcal{D}, u)$ is higher than when $h^F_w$ is forced to $\theta$ in the RHS of Eq. (10) and thus:

$$\psi^P(\mathcal{D}, u) \geq W_h(\mathcal{L} - t^P_h - \mathcal{D}) + W_P \theta - \tilde{C}(t^P_h, \mathcal{L} - \theta, u).$$  \hspace{1cm} (11)$$

Using Eq. (11),

$$\psi^P(\mathcal{D}, u) - \psi^F(\mathcal{D}, u) \geq W_h(t^F_h - t^P_h) + [(W_P \theta - W_F h^F_w) + \tilde{C}(t^F_h, \mathcal{L} - h^F_w - \mathcal{D}, u) - \tilde{C}(t^P_h, \mathcal{L} - \theta, u)]$$

$$= (W_P \theta - W_F h^F_w) + \tilde{C}(t^F_h, \mathcal{L} - h^F_w - \mathcal{D}, u) - \tilde{C}(t^P_h, \mathcal{L} - \theta, u).$$  \hspace{1cm} (12)$$

The last equality follows because

$$\tilde{C}(t^F_h, \mathcal{L} - h^F_w - \mathcal{D}, u) - \tilde{C}(t^P_h, \mathcal{L} - h^F_w - \mathcal{D}, u) = \frac{\partial \tilde{C}(\cdot)}{\partial t_h}(t^F_h - t^P_h),$$

and $W_h + \partial \tilde{C}(\cdot)/\partial t_h = 0$. 
Let $\Delta h_w = h_w^F - \theta \geq 0$. Then the last expression of Eq. (12) becomes

$$W_P\theta - W_F(\theta + \Delta h_w) + \tilde{C}(l_h^P, \overline{L} - (\theta + \Delta h_w) - \overline{D}, u) - \tilde{C}(l_h^P, \overline{L} - \theta, u). \quad (13)$$

Because $W_F = -\partial \tilde{C}/\partial l_w$, the following relationship holds:

$$-W_F \Delta h_w + \tilde{C}(l_h^P, \overline{L} - (\theta + \Delta h_w) - \overline{D}, u) - \tilde{C}(l_h^P, \overline{L} - \theta - \overline{D}, u) = 0. \quad (14)$$

Using Eq. (14), Eq. (13) becomes $(W_P - W_F)\theta + \Delta C(l_h^P, u)$. Assumption 1 implies this is positive. Then $\psi^P(D, u) > \psi^F(D, u)$ follows.

The maximum of $\psi^P(D, u)$ and $\psi^F(D, u)$, $\psi(D, u)$, gives the bid-rent function for a household when the wife has options to work either full time or part time.

For households for which Assumption 1 holds, $\psi^P(D, u) > \psi^F(D, u)$ and $\psi^P(0, u) < \psi^F(0, u)$, and therefore, $\psi^P(.)$ and $\psi^F(.)$ intersect once.\textsuperscript{14} Whereas $\psi^P(0, u) < \psi^F(0, u)$ is true for all households, $\psi^P(D, u) > \psi^F(D, u)$ requires an assumption. However, if $\psi^P(D, u) \leq \psi^F(D, u)$ for all, the wife works part time in no households. Since part-time work has become increasingly common in Japan in recent decades, it is natural to expect that this type of assumption holds.

To facilitate the following discussion, two types of reservation wage are introduced.

\textsuperscript{14}Since the bid-rent gradient is constant in this model, they do not intersect more than once.
Definition 1 (Reservation wage for part-time work). The wife’s reservation wage for part-time work, $w^*_P$, satisfies the following condition:

$$w^*_P(h, D, u) = -\frac{\partial \tilde{C}}{\partial l_w}(\mathcal{L} - h - D, \mathcal{L}, u).$$

The reservation wage for part-time work (measured in consumption goods) is similar to the standard reservation wage concept determining whether an individual works or not.

Definition 2 (Reservation wage for full-time work). The wife’s reservation wage for full-time work, $w^*_F$ is

$$w^*_F(h, D, u) = -\frac{\partial \tilde{C}}{\partial l_w}(\mathcal{L} - h - D, \mathcal{L} - \theta - D, u).$$

The four modes of work are illustrated in Figure 4 when the utility function is $U(l_h, l_w, C) = u^h(l_h) + u^w(l_w) + v(C)$. The horizontal axis represents the wife’s leisure time ($l_w$), and the vertical axis measures the marginal utility of her leisure ($\partial u^w/\partial l_w$). I make the following simplifying assumptions on the shape of the utility function.

Assumption 2. The utility function, $U$, is concave, and is separable between $l_h$ and $C$ and between $l_w$ and $C$, so that $U_{jC} = 0$ (j = h, w). Assume further that $U_{wh} \leq 0$.

The assumption that $U_{wh} \leq 0$ is likely to hold if household responsibilities are shared by both spouses. With Assumption 2, the following proposition
gives an interesting property of the above model.

**Proposition 2.** Under Assumption 2,

\[
\frac{d\hat{w}_F^*}{dD} > 0.
\]

**Proof.** By the definition of the reservation wage for the wife’s full-time work and by the relation satisfied on the indifference surface,

\[
w_F^*(h, D, u) = -\frac{\partial \tilde{C}(\_)}{\partial l_w} = \frac{U_w(\bar{L} - h_h - D, \bar{L} - h_w^F - D, \hat{C}(\bar{L} - h_h - D, \bar{L} - h_w^F - D, u))}{U_C(\bar{L} - h_h - D, \bar{L} - h_w^F - D, \hat{C}(\bar{L} - h_h - D, \bar{L} - h_w^F - D, u))},
\]

(15)

Differentiating Eq. (15) with respect to \(D\) and using \(U_{jC} = 0\) \((j = h, w)\), I obtain

\[
\frac{\partial}{\partial D}(-\frac{\partial \tilde{C}}{\partial l_w}) = \frac{(-U_{wh} - U_{ww})U_C + U_{CC}(\hat{C}_h + \hat{C}_w)U_w}{U_C^2},
\]

where \(\hat{C}_j = \partial \hat{C}/\partial l_j(<0)\). Because concavity of \(U\) implies \(U_{CC} < 0, U_{ww} < 0,\) and \(U_{wh} \leq 0\) by assumption, it follows that \(\frac{\partial}{\partial D}(-\frac{\partial \tilde{C}}{\partial l_w}) > 0\).

Proposition 2 says that when each spouse’s leisure and consumption are separable and \(U_{wh} \leq 0\), closer location to the CBD reduces the wife’s reser-
vation wage for full-time work. As a result, the likelihood that the wife works full time increases when the household locates close to the CBD, other things equal. The shorter commuting time decreases the fixed cost of full-time work for the wife. Proposition 2 implies that the wages of full-time working wives living closer to the CBD are not necessarily higher than those of wives living farther from the CBD. On the one hand, a household in which the wife works full time has more incentive to locate close to the CBD due to the AMM condition (e.g., Iwata and Tamada 2008). On the other hand, as shown in Proposition 2, it is easier for wives located close to the CBD for other reasons to participate in full-time work.

Once the wife participates as a full-time worker, a higher $W_F$ increases her full-time hours. This serves to reinforce the discrete nature of the labor market because the earnings differential between full-time working women and others widens. This point is summarized in the following proposition.

**Proposition 3.** Consider a change in full-time wage, $W_F$, in the range $w^*_F \leq W_F$. Then:

$$-\frac{\partial \psi^F_D}{\partial W_F} > 0,$$

where $\psi^F_D$ is the bid-rent gradient. If the substitution effect dominates the income effect in the wife’s leisure choice,

$$dh_w/dW_F > 0.$$

**Proof.** When the wife works full time, the bid-rent gradient equals the summed
wages of the husband and wife. $-\partial \psi^F_D / \partial W_F > 0$ implies that $D$ decreases with a higher $W_F$. From the first-order condition for the choice of hours (Eq. (5)), $h_w + D$ increases with $W_F$ as long as the substitution effect dominates the income effect. Combining these two results, $h_w$ increases unambiguously with $W_F$.

Note that a high $W_F$ has two reinforcing impacts on $h_w$. First, a higher wage decreases the amount of leisure, as in the standard labor supply model. The second impact is to reduce $D$ by locating closer to the CBD. A lower value of $D$ allows a full-time working wife to increase $h_w$ further. A similar argument applies to the husband’s hours, $h_h$.

The distribution of $W_h + I(h_w > \theta) \cdot W_F$ is determined by the correlation between the husband’s wage and the wife’s full-time wage, as well as the correlation between the husband’s wage and the likelihood that the wife works full time.\footnote{Empirical studies in Japan to date have not reached a consensus regarding the sign of the correlation between the husband’s wage and the wife’s labor force participation. In the past this correlation was thought to be negative and some studies using recent data still find a negative correlation (e.g., Sasaki 2002; Nawata and Li 2004). However, other studies point out the possibility that this correlation has become weaker over time, and may even have turned positive (e.g., Kohara 2008).} An implication is that housing units located close to the CBD are more likely to be occupied by two-earner households.

### 3.2 Extensions

The basic model above did not consider explicitly the link between household characteristics and the shape of the utility function. Observable household...
characteristics, such as the number of children or children’s ages, are likely to be related to the shape of the utility function. The number of children, especially of young children, is likely to increase the marginal value of the wife’s leisure time. Furthermore, larger household size increases the demand for space. Statistics of the Tokyo Metropolitan Area clearly show that the fertility rate in 2006 was higher in the suburban prefectures than in Tokyo: the total fertility rate was 1.02 in Tokyo and approximately 1.23 for Saitama, Chiba, and Kanagawa.16

Suppose there are two types of households, those with children and those without. The bid-rent for households with children takes higher values (for any $D$) than for childless households for two reasons. First, since the households with children have a greater demand for space, the bid-rent per space is lower for households with children than that for childless households, other things equal. Second, in households with children, the marginal utility from the wife’s leisure time is higher than otherwise, which makes the wife less likely to participate either in full-time or part-time employment. This, in turn, makes the bid-rent of households with children lower than that of childless households because the wife’s earnings are lower.17 Therefore, the presence of children makes the division between the two employment options

16The figures are from the Long-term Population Statistics of Japan.
17Note that the wife’s earnings appear in the objective functions for deriving the bid-rent curves (such as Eqs. (2) and (3)).
for wives more distinct than in the basic model laid out in Section 3.1.\textsuperscript{18}

4 A numerical example

This section presents a numerical example of the basic model in Section 3.1. The utility function is assumed to take the following form:

$$U(l_h, l_w, C) = \alpha \ln(l_h) + \beta \ln(l_w) + C. \quad (16)$$

I assume $\alpha < \beta = 0.6$. It is assumed that the full-time wages of husbands and wives, $W_h$ and $W_F$, follow uniform distributions: $W_h$ has a uniform distribution over the interval $[1,4]$, and $W_F$ has a uniform distribution over the interval $[0.5,1.5]$. Note that the mean full-time wage is higher for males than for females, and that the variance of full-time wage is larger for males than for females. It is assumed that the wife’s part-time wage, $W_P$, is correlated with $W_F$ as follows:

$$W_P = \begin{cases} 
0.75 \times W_F & \text{if } W_F \leq W_F^{25th} \\
0.7 \times W_F & \text{if } W_F^{25th} < W_F \leq W_F^{med} \\
0.7 \times W_F^{med} & \text{if } W_F > W_F^{med} 
\end{cases}. \quad (17)$$

\textsuperscript{18}Beckman (1973) points out that single persons and childless couples locate closer to the CBD whereas large families live in suburbs. In his model, the labor supply choices are not endogenous.
For wives whose full-time wages are lower than the 25th percentile of the full-time wage distribution, the wage gap between full-time and part-time jobs is 25 percent. For wives whose full-time wages are between the 25th percentile and the median, this gap is 30 percent. For wives with full-time wages above the median, the part-time wage is 70 percent of the median full-time wage. These assumptions reflect the empirical facts that the part-time/full-time wage gap for female workers is smaller for the less educated and that part-time wages do not increase much with education.

Note that I normalize the lower bound of the husband’s full-time wage to 1. For simplicity, the matching between husband and wife is assumed to be random in terms of their full-time wage levels.\(^{19}\) The total time endowment \(L\) is normalized to 1; \(D, \theta, h_w,\) and \(h_h\) are measured using this unit. For instance, if the total time endowment is 80 hours per week, a 1-hour commute each way costs 10 hours per week (assuming full-time employees work 5 days per week) so \(D\) is 0.125 (=10/80). The threshold \(\theta\) for full-time work is set to 0.3125 (25 hours out of 80). A piecewise linear relationship between \(-\psi_D\)

\(^{19}\)An example with simple assortative matching between spouses is also constructed, but the results are not reported here. The properties of the solution are essentially the same.
and the $D$ is assumed, as follows:

$$D(-\psi_D) = \begin{cases} 
-(1/24)(-\psi_D - 1) + D(1) & \text{if } 1 \leq -\psi_D < 1.3 \\
-(1/32)(-\psi_D - 1.3) + D(1.3) & \text{if } 1.3 \leq -\psi_D < 2.5 \\
-(1/40)(-\psi_D - 2.5) + D(2.5) & \text{if } 2.5 \leq -\psi_D \leq 5.5 
\end{cases} \quad (18)$$

where $D(-\psi_D)$ corresponds to the $D$ value chosen by households for which the bid-rent gradient is $\psi_D$. The model is solved numerically by making discrete approximations to the distributions of wages and $-\psi_D$.

The proportions of wives working part time and full time are derived from the above model, as shown in Figure 5.A. In households located closest to the CBD, all wives work full time. In other words, the housing units closest to the CBD are occupied only by two-earner households with both spouses working full time. The proportion of wives working full time decreases as $D$ increases and eventually reaches zero. The proportion of wives working part time is zero for the lowest $D$ values, but increases as $D$ increases. Although the behaviors described are extreme, they are also consistent with the empirical patterns shown in Figure 1.A.

The relationship between wage levels and commuting time of working wives is shown in Figure 5.B. The size of the points/circles in Figure 5.B indicates the number of wives working at that particular combination of wage levels.

---

20In a more fundamental sense, this relationship should be derived by solving for the market clearing conditions of the housing market. I instead assume that the supply of housing units at each location $D$ yields this relationship.
By construction, the commuting time for part-time workers is zero, and part-time wages have a smaller dispersion than full-time wages. As shown in Figure 5.B, in equilibrium, some full-time working wives with low wages also have low values of \( D \). This is because lower values of \( D \) reduce the reservation wage for full-time work (Proposition 2). Since part-time wages are less variable and the commuting time is zero for all part-time workers, the outcomes for these workers are compressed into the lower left corner as mass points. These results are quite different from the relationship found by Iwata and Tamada (2008), who find that working wives with the highest wages live closest to the CBD. Figure 5.B shows that some full-time working wives with high wages also have high \( D \) values.

Although the example here is quite specific in that full-time wages are assumed to follow uniform distributions and that matching between husband and wife is random, it is consistent with the empirical features of the education and housing location distributions (Figures 1-3).

5 Conclusions

In this paper, a simple model of family labor supply and residential location choice is presented. The model incorporates an important institutional feature of the Japanese labor market: the woman’s choice between full-time work and part-time work. It can explain why women’s participation behav-

\[ \text{\footnotesize \textsuperscript{21}} \] For the upper-right section of the figure, the size of individual circles is very small and the distances between the circles are short, so that they are collectively seen as thick lines.
iors differ significantly in areas that are geographically close but differ in real estate prices. Women who earn high full-time wages in the CBD tend to live close to the workplace, whereas those with lower wages live in suburbs where housing prices are lower. The model also explains that high commuting costs (or equivalently, high housing prices in locations close to the CBD) could be one of the main obstacles to women’s full-time employment in the Tokyo Metropolitan Area. In Japan, full-time and part-time employment are two distinct labor supply choices for women. I argue that the housing market operates in a way to further strengthen this discrete feature in the labor market.

One notable change in the Japanese labor market over the last two decades has been an increase in female wages: from 1990 to 2000, the average hourly wage of female regular workers rose by 20.8 percent, while that of male regular workers rose by 8.7 percent (in real terms).\(^\text{22}\) The labor force participation rate of married women aged 25-54 increased from 53 percent in 1980 to 59 percent in 2000.\(^\text{23}\) An investigation of how these changes in the labor market have affected the housing market and the residential patterns is left for future research.

\(^{22}\) These are derived from the Basic Survey of Wage Structure (Ministry of Health, Welfare and Labour of Japan) and CPI statistics (Ministry of Internal Affairs and Communications of Japan).

\(^{23}\) Labour Force Survey (Statistics Bureau, Ministry of Internal Affairs and Communications of Japan).
References


Mok, Diana (2007) “Do two-earner households base their choice of residential location on both incomes?” Urban Studies 44, 723-750.


Takeishi, Emiko (2007) Transitions of women’s career and inter-regional comparison [“Macro data de miru jyosei no career no hensen to chiiki kan hikaku”] Shogai gakushu to career design 4, Hosei University, 19-34 (in Japanese).


Table 1: Proportion of power men and women and housing price

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<td></td>
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<td>working</td>
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<td>wage-salary earner</td>
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</tr>
<tr>
<td>Men</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Women</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Log(land price)</strong></td>
<td>0.188 **</td>
<td>0.101 **</td>
<td>0.163 **</td>
<td>0.088 **</td>
</tr>
<tr>
<td></td>
<td>(0.038)</td>
<td>(0.018)</td>
<td>(0.034)</td>
<td>(0.016)</td>
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<tr>
<td>Dummy for Age 30-34</td>
<td>0.031</td>
<td>-0.066 **</td>
<td>0.021</td>
<td>-0.071 **</td>
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<td>(0.043)</td>
<td>(0.020)</td>
<td>(0.039)</td>
<td>(0.018)</td>
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<tr>
<td>Dummy for Age 35-39</td>
<td>0.082</td>
<td>-0.089 **</td>
<td>0.059</td>
<td>-0.100 **</td>
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<td>(0.044)</td>
<td>(0.021)</td>
<td>(0.040)</td>
<td>(0.019)</td>
</tr>
<tr>
<td>Dummy for Age 40-44</td>
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<td>-0.079 **</td>
<td>0.073</td>
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<td>(0.047)</td>
<td>(0.022)</td>
<td>(0.043)</td>
<td>(0.020)</td>
</tr>
<tr>
<td>Dummy for Age 45-49</td>
<td>0.046</td>
<td>-0.108 **</td>
<td>0.004</td>
<td>-0.122 **</td>
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<tr>
<td></td>
<td>(0.047)</td>
<td>(0.022)</td>
<td>(0.042)</td>
<td>(0.020)</td>
</tr>
<tr>
<td>Dummy for Age 50-54</td>
<td>-0.008</td>
<td>-0.141 **</td>
<td>-0.050</td>
<td>-0.151 **</td>
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<tr>
<td></td>
<td>(0.044)</td>
<td>(0.020)</td>
<td>(0.040)</td>
<td>(0.019)</td>
</tr>
<tr>
<td>Dummy for Age 55-59</td>
<td>-0.086</td>
<td>-0.176 **</td>
<td>-0.121 **</td>
<td>-0.181 **</td>
</tr>
<tr>
<td></td>
<td>(0.046)</td>
<td>(0.021)</td>
<td>(0.042)</td>
<td>(0.019)</td>
</tr>
<tr>
<td>Observations</td>
<td>56</td>
<td>56</td>
<td>56</td>
<td>56</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.49</td>
<td>0.72</td>
<td>0.51</td>
<td>0.76</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors in parentheses.
The base group for age group is age 25-29.
A constant is included in each regression, but the coefficients are not shown.
* significant at the 5% level; ** significant at the 1% level.

Table 2: Labor force participation rate and housing price

<table>
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<tr>
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<th>(1) University</th>
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<th>(3) University</th>
<th>(4) Senior High</th>
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<tr>
<td><strong>Women</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log(land price)</td>
<td>0.012</td>
<td>0.053**</td>
<td>-0.001</td>
<td>0.004</td>
</tr>
<tr>
<td>(0.015)</td>
<td>(0.011)</td>
<td>(0.001)</td>
<td>(0.002)</td>
<td></td>
</tr>
<tr>
<td>Dummy for Age 30-34</td>
<td>-0.171**</td>
<td>-0.122**</td>
<td>0.015**</td>
<td>0.008*</td>
</tr>
<tr>
<td>(0.015)</td>
<td>(0.018)</td>
<td>(0.003)</td>
<td>(0.004)</td>
<td></td>
</tr>
<tr>
<td>Dummy for Age 35-39</td>
<td>-0.250**</td>
<td>-0.119**</td>
<td>0.019**</td>
<td>0.011**</td>
</tr>
<tr>
<td>(0.013)</td>
<td>(0.014)</td>
<td>(0.003)</td>
<td>(0.004)</td>
<td></td>
</tr>
<tr>
<td>Dummy for Age 40-44</td>
<td>-0.246**</td>
<td>-0.059**</td>
<td>0.020**</td>
<td>0.011**</td>
</tr>
<tr>
<td>(0.014)</td>
<td>(0.014)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td></td>
</tr>
<tr>
<td>Dummy for Age 45-49</td>
<td>-0.241**</td>
<td>-0.028</td>
<td>0.018**</td>
<td>0.011**</td>
</tr>
<tr>
<td>(0.019)</td>
<td>(0.015)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td></td>
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<tr>
<td>Dummy for Age 50-54</td>
<td>-0.256**</td>
<td>-0.044**</td>
<td>0.014**</td>
<td>0.007*</td>
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<tr>
<td>(0.023)</td>
<td>(0.016)</td>
<td>(0.003)</td>
<td>(0.004)</td>
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<tr>
<td>Dummy for Age 55-59</td>
<td>-0.331**</td>
<td>-0.109**</td>
<td>-0.002</td>
<td>-0.006</td>
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<tr>
<td>(0.023)</td>
<td>(0.019)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td></td>
</tr>
</tbody>
</table>

| **Men**              |               |                 |               |                 |
| Observations         | 56            | 56              | 56            | 56              |
| R-squared            | 0.89          | 0.73            | 0.84          | 0.57            |

Notes: Robust standard errors in parentheses.
The base group for age group is age 25-29.
A constant is included in each regression, but the coefficients are not shown.
* significant at the 5% level; ** significant at the 1% level.
Figure 1.A: Employment population ratios, regular employment ratios, and part-time employment ratios of women in four prefectures in Tokyo MA.

Source: Employment Status Survey 2007
Figure 1.B: Employment population ratios and regular employment ratios of men in four prefectures in Tokyo MA

Source: Employment Status Survey 2007
Figure 2: Share of residents with a university degree or more, by prefecture in Tokyo Metropolitan Area

Share of residents with a university degree or more by Prefecture: Men

Share of residents with a university degree or more by Prefecture: Women

Figure 3: Share of power men & women among residents by prefecture, Tokyo Metropolitan Area

Proportion of power men among residents

Proportion of power women among residents

Figure 4: Wife’s employment choice with a separable utility function:

\[ U(l_h, l_w, C) = u^h(l_h) + u^w(l_w) + v(C) \]
Figure 5.A: Distribution of wife’s employment status and D

Figure 5.B: Working wives' commuting time and wage

Note: Size of the circle corresponds to the number of wives with the corresponding commuting time and wage. For the upper-right section, the size of individual circles is very small and the distances between the circles are short, so that they are collectively seen like thick lines.