マクロ及びミクロ経済学
出題の趣旨・解答例

Micro and Macroeconomics
Solutions

1.

$$\frac{Y}{L} = A\left(\frac{K}{L}\right)^{\alpha}\left(\frac{H}{L}\right)^{\beta}$$
$$y = Ak^{\alpha}h^{\beta}$$
$$\frac{y_2}{y_1} = \frac{A_2}{A_1}\left(\frac{k_2}{k_1}\right)^{\alpha}\left(\frac{h_2}{h_1}\right)^{\beta}$$
$$\log\left(\frac{y_2}{y_1}\right) = \log\left(\frac{A_2}{A_1}\right) + \alpha\log\left(\frac{k_2}{k_1}\right) + \beta\log\left(\frac{h_2}{h_1}\right).$$

Because $\log(A_2/A_1)$, $\log(k_2/k_1)$, and $\log(h_2/h_1)$ each increase by 1%, $\log(y_2/y_1)$ should increase by $(1+\alpha+\beta)$%.

2. (1) X's profit maximization problem is $\max_y yD(y) - C(y)$, so the first-order condition is
$yD'(y) + D(y) - C'(y) = 0$
and the second-order condition is
$yD''(y) + 2D'(y) - C''(y) < 0.$
In our specification, each becomes

$$y = \left(\frac{a(1-b)}{c}\right)^{\frac{1}{b+1}}$$
$$-ab(1-b)y^{-b-1} - c < 0.$$

The SOC always holds because the left-hand side is negative. Hence the equilibrium quantity is given by the above.

(2) X's profit maximization problem is $\max_y py - C(y)$, for which the FOC is $p - C'(y) = 0$ and the SOC is $-C''(y) < 0$. The SOC always holds. Combining the FOC and the equilibrium condition $p = D(y)$, we have

$D(y) = C'(y)$, or

$$y = \left(\frac{a}{c}\right)^{\frac{1}{b+1}}.$$

Because $b \in (0,1)$, the monopolist produces less than the competitive firm. As $b \to 0$ (that is, when the demand elasticity is infinite), the monopolistic supply and the competitive supply coincide.

3. Because $x = \sigma z + \mu$, where $z \sim N(0,1)$, we have

$$\begin{aligned} E[u(x)] &= \int_{-\infty}^{\infty} -e^{-\rho(\sigma z+\mu)} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \\ &= \int_{-\infty}^{\infty} -\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z+\rho\sigma)^2 + \frac{\rho^2\sigma^2}{2} - \rho\mu} dz \\ &= -e^{\frac{\rho^2\sigma^2}{2} - \rho\mu} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z+\rho\sigma)^2} dz \\ &= -e^{-\rho(\mu - \frac{\rho\sigma^2}{2})} \\ &= u(\mu - \frac{\rho\sigma^2}{2}), \end{aligned}$$

where the second from the last equality is because $\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z+\rho\sigma)^2}$ is the density function of $N(-\rho\sigma, 1)$ and hence $\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z+\rho\sigma)^2} = 1$. Thus the certainty equivalent is $\mu - \frac{\rho\sigma^2}{2}$.

4. (1) ${}_3C_2 + 1 = 4$. The first three correspond to pure-strategy equilibria in which exactly two players choose to act and the public goal is achieved. There is also an equilibrium in which no one acts.

(2) Let's say one of the three players is Kate. Suppose the other two players follow a mixed strategy in which they choose 'Act' with probability p and 'Not act' with probability $1-p$. Then, (i) Kate's expected payoff from choosing 'Act' is $-C(1-p)^2 + (1-C)[2p(1-p) + p^2]$. Also, (ii) her expected payoff from choosing 'Not act' is p^2. (i) $-$ (ii) $= 0$ gives

$$2p^2 - 2p + C = 0,$$

or

$$\begin{aligned} p &= \frac{1 \pm \sqrt{1-2C}}{2} \\ &= 0.1, 0.9. \end{aligned}$$

That is, in one symmetric equilibrium, everyone acts with probability 0.1. In the other symmetric equilibrium, everyone acts with probability 0.9.

5. (1) First, derive the reaction function for each firm. The profit maximization problem for firm 1 is:

$$\max \pi_1 = (a - q_1 - q_2 - c_1)q_1$$

FOC gives $a - q_2 - c_1 = 2q_1$. Similarly, the reaction function for firm 2 is $a - q_1 - c_2 = 2q_2$. By solving these two equations, $q_1^* = \frac{a+c_2-2c_1}{3}$, $q_2^* = \frac{a+c_1-2c_2}{3}$, and thus $Q^* = q_1^* + q_2^* = \frac{2a-c_1-c_2}{3}$. Equilibrium price is obtained by substituting Q^* into the inverse demand function. Therefore, $P^* = \frac{a+c_1+c_2}{3}$.

(2) Firm 1's profit is $\pi_1^* = \frac{(a+c_2-2c_1)^2}{9}$. Firm 2's profit is $\pi_2^* = \frac{(a+c_1-2c_2)^2}{9}$. Consumer surplus is $CS^* = (a - P^*) \times Q^* \times \frac{1}{2} = \frac{(2a-c_1-c_2)^2}{18}$. Producer surplus is $PS^* = \pi_1^* + \pi_2^* = \frac{(a+c_2-2c_1)^2}{9} + \frac{(a+c_1-2c_2)^2}{9}$.

6. (1) Making a conjecture that the constraint should hold with equality, the Lagrangian is:

$$\mathcal{L} = rK + wL + \lambda(\bar{y} - zK^a L^{1-a}).$$

The optimal choice (K^*, L^*, λ) is the solution to the following FOCs:

$$\begin{aligned} r &= \lambda z a (K^*/L^*)^{a-1} & (1) \\ w &= \lambda z (1-a)(K^*/L^*)^a & (2) \\ \bar{y} &= z(K^*)^a (L^*)^{1-a}. & (3) \end{aligned}$$

(2) Dividing (2) by (1) yields

$$\frac{K^*}{L^*} = \frac{a}{1-a} \cdot \frac{w}{r}.$$

Use the constraint to find factor demands:

$$\begin{aligned}\bar{y} &= z(K^*)^a(L^*)^{1-a} \\ &= z\left(\frac{K^*}{L^*}\right)^a L^* \\ &= z\left(\frac{a}{1-a}\cdot\frac{w}{r}\right)^a L^*,\end{aligned}$$

or

$$L^* = \bar{y}z^{-1}\left(\frac{1-a}{a}\cdot\frac{r}{w}\right)^a.$$

Similarly, we have

$$K^* = \bar{y}z^{-1}\left(\frac{a}{1-a}\cdot\frac{w}{r}\right)^{1-a}$$

Also, by (1),

$$\begin{aligned}\lambda &= r(za)^{-1}(K^*/L^*)^{1-a} \\ &= r(za)^{-1}\left(\frac{a}{1-a}\cdot\frac{w}{r}\right)^{1-a} \\ &= z^{-1}\left(\frac{r}{a}\right)^a\left(\frac{w}{1-a}\right)^{1-a}.\end{aligned}$$

Thus we have obtained the solution (K^*, L^*, λ).

(3) First, calculating $K^*\times(1)+L^*\times(2)$ gives the optimized objective:

$$\begin{aligned}rK^* + wL^* &= \lambda za(K^*)^a(L^*)^{1-a} + \lambda z(1-a)(K^*)^a(L^*)^{1-a} \\ &= \lambda a\bar{y} + \lambda(1-a)\bar{y} \\ &= \lambda\bar{y}.\end{aligned}$$

Second, notice that λ depends on many parameters but not on $\bar{y}$. These two together imply that the cost function is linear in the amount of output (i.e., the marginal cost is constant).

7. (1) $\bar{b}$ is obtained by $\frac{\bar{b}v_1}{6} - p_1 = \frac{\bar{b}v_2}{6} - p_2 \Leftrightarrow \bar{b} = 6\frac{p_2-p_1}{v_2-v_1}$. Because b is distributed uniformly on $[2, 8]$ and the number of consumers is 6, demand for firm 1 is $q_1(p_1, p_2) = \bar{b} - 2 = 6\frac{p_2-p_1}{v_2-v_1} - 2$. Similarly, demand for firm 2 is $q_2(p_1, p_2) = 8 - \bar{b} = 8 - 6\frac{p_2-p_1}{v_2-v_1}$.

(2) Firm 1's maximization problem is:

$$\max \pi_1(p_1, p_2) = (p_1 - cv_1)\left(6\frac{p_2 - p_1}{v_2 - v_1} - 2\right)$$

FOC with respect to p_1 gives $p_1 = \frac{p_2+cv_1}{2} - \frac{v_2-v_1}{6}$. A similar problem to firm 2 gives $p_2 = \frac{p_1+cv_2}{2} + \frac{2(v_2-v_1)}{3}$. Solving these two equations gives equilibrium prices: $p_1^* = \frac{c(v_2+2v_1)}{3} + \frac{2(v_2-v_1)}{9}$ and $p_2^* = \frac{c(2v_2+v_1)}{3} + \frac{7(v_2-v_1)}{9}$. Equilibrium profits are $\pi_1^* = \frac{2(v_2-v_1)}{3}\left[c + \frac{2}{3}\right]^2$ and $\pi_2^* = \frac{2(v_2-v_1)}{3}\left[\frac{7}{3} - c\right]^2$.

(3) No. If firm 1 increases their quality v_1, the price competition becomes tougher and therefore its equilibrium profit ($\pi_1^* = \frac{2(v_2-v_1)}{3}\left[c + \frac{2}{3}\right]^2$) becomes smaller.

経済思想

出題の趣旨・解答例

問題Ⅰ．　経済思想に関する基本的な知識を問う問題である。カール・マルクスは経済学のみならず、この世界の在り方そのものに大きな影響を与えてきた。思想としては、『資本論』のほかに、『経済学・哲学草稿』などの初期の論稿が重要な文献となる。資本主義と社会主義（共産主義）の体制選択問題や、現代の資本主義社会をいかに改革するかという問題に、マルクスの思想をどのように活かすことができるのか、あるいはできないのか。本設問は、マルクスの理解と評価に関する基本的な知見を問うている。

問題Ⅱ．　現代の経済思想に関する基本的な質問である。センの学問的功績は、社会的選択理論・厚生経済学・開発経済学・現代正義論など多岐に及ぶ。彼の制度論的なテーマは、貧困問題や開発問題における資源配分方法の仕組みの解明と、社会選択プロセスの解明である。それを支える理念的なテーマは、福祉理念や自由の理念への分析である。前者の観点から、センは近代経済学の利益追求システムを批判的に検討し、そのシステムへの依存を飢餓の原因と位置づけつつ、それを改革するための民主的手続きとして社会選択プロセスの解明を試みる。また後者の観点から、近代経済学のシステムの基礎となる合理性の仮定を批判的に検討し、個人は自己利益のみを追求するのではなく、自らの目的自体を検討する合理性と自由をもつべきであると唱えた。そこから、センは自由の指標として潜在能力アプローチを提唱した。本設問においては、このような多岐に及ぶセンの学問的功績に対して、センの理論的分析や規範的理念の特徴を理解しているかが問われる。

統計学

問題 I（解答例）

1. 他の条件を一定にした時, 飲食店でタバコが禁止されている地域では, 喫煙量が平均して 2.825 本少なくなる.

2. $t = -0.501/0.167 = 3$.

3. $0.771 - 2 * 0.009 age < 0$ を解いて, $age > 42.8$.

4. 不均一分散下では推定量は線形不偏推定量となり, 線形モデルの中で最も効率の良い推定量ではなくなる.

5. 下記のような検定が考えられる.

   - Breusch-Pagan (BP) test
     (a) 残差を計算し $\hat{u}_i$ を得る.
     (b) $\hat{u}^2 = \gamma_0 + \gamma_1 lincome + \gamma_2 lcigpric + \gamma_3 educ + \gamma_4 age + \gamma_5 agesq + \gamma_6 restaurn + \epsilon$ を OLS 推定する.
     (c) H_0: 全ての係数（切片を除く）が同時に *0* を F 検定/LM 検定.
   - White test
     (a) 残差を計算し $\hat{u}_i$ を得る.
     (b) $\hat{u}^2 = \gamma_0 + \gamma_1 lincome + \gamma_2 lcigpric + \gamma_3 educ + \gamma_4 age + \gamma_5 agesq + \gamma_6 restaurn + interactions + polynomials + \epsilon$ を OLS 推定する.
     (c) H_0: 全ての係数（切片を除く）が同時に *0* を F 検定/LM 検定.
   - White test (special version)
     (a) 残差を計算し $\hat{u}_i$ を得る.
     (b) 予測値 $\hat{y}_i$ を得る.
     (c) $\hat{u}^2 = \delta_0 + \delta_1 \hat{y} + \delta_2 \hat{y}^2 + \epsilon$ を OLS 推定する.
     (d) H_0: 全ての係数（切片を除く）が同時に *0* を F 検定/LM 検定.

6. 各観測値に $w_i = \frac{1}{\sqrt{lincome_i}}$ で重みづけを行ない,OLS 推定を行なう. このとき, 誤差項は均一分散を持つため, 推定量は最良線形不偏推定量 (BLUE) の性質をもつ.

7. (1) 説明変数と誤差項の間に相関がある場合を指す.

(2) このとき OLS 推定量は一致性を失う.

(3) 操作変数法.$educ$ と相関を持つが, 誤差項と相関を持たないような操作変数 z を準備する.z によって説明される $educ$ の外生的な変動部分を説明変数に利用して推定を行なう (2SLS).

1. When other conditions are held constant, in areas where smoking is prohibited in restaurants, the average smoking quantity decreases by 2.825 cigarettes.

2. $t = -0.501/0.167 = 3$.

3. Solve $0.771 - 2 * 0.009 * age < 0$ to get $age > 42.8$.

4. Under heterosckdasticity, the estimator becomes a linear unbiased estimator, no longer being the most efficient estimator within the linear model.

5. The following tests are considered:

   - Breusch-Pagan (BP) test
     (a) Calculate the residuals to obtain $\hat{u}_i$.
     (b) Estimate $\hat{u}^2 = \gamma_0 + \gamma_1 lincome + \gamma_2 lcigpric + \gamma_3 educ + \gamma_4 age + \gamma_5 agesq + \gamma_6 restaurn + \epsilon$, using OLS.
     (c) Test H_0: All coefficients excluding an intercept are jointly equal to 0, using an F-test/LM-test.
   - White test
     (a) Calculate the residuals to obtain $\hat{u}_i$.
     (b) Estimate $\hat{u}^2 = \gamma_0 + \gamma_1 lincome + \gamma_2 lcigpric + \gamma_3 educ + \gamma_4 age + \gamma_5 agesq + \gamma_6 restaurn + \text{interactions} + \text{polynomials} + \epsilon$, using OLS.
     (c) Test H_0: All coefficients excluding an intercept are jointly equal to 0, using an F-test/LM-test.
   - White test (special version)
     (a) Calculate the residuals to obtain $\hat{u}_i$.
     (b) Obtain predicted values $\hat{y}_i$.
     (c) Estimate $\hat{u}^2 = \delta_0 + \delta_1 \hat{y} + \delta_2 \hat{y}^2 + \epsilon$, using OLS.

(d) Test H_0: All coefficients excluding an intercept are jointly equal to 0, using an F-test/LM-test.

6. We perform OLS estimation with each observation weighted by $w_i = \frac{1}{\sqrt{lincome_i}}$. In this case, the error term has homoskedasticity, and the estimator possesses the properties of the Best Linear Unbiased Estimator (BLUE).

7. (1) It refers to a situation where there is correlation between the explanatory variables and the error term.

   (2) In such cases, the OLS estimators lose consistency.

   (3) Instrumental Variable (IV) method can deal with the endogeneity. We prepare an instrumental variable z that is correlated with *educ* but uncorrelated with the error term. Then, we use the exogenous variation of *educ* explained by z as an explanatory variable in the estimation (2SLS).

問題 II（解答例）

1. $\mathbb{E}[X_i]$ and $\mathbb{V}[X_i]$ are,

$$\mathbb{E}[X_i] = p, \quad \mathbb{V}[X_i] = p(1-p).$$

2. $\mathbb{E}\left[\bar{X}\right]$ and $\mathbb{V}\left[\bar{X}\right]$ are,

$$\mathbb{E}\left[\bar{X}\right] = p, \quad \mathbb{V}\left[\bar{X}\right] = \frac{p(1-p)}{n}.$$

3. When the sample size is large, the asymptotic distribution of the random variable $\frac{\bar{X}-p}{\sqrt{p(1-p)/n}}$ is the standard normal distribution and that of the random variable $\frac{n\left(\bar{X}-p\right)^2}{p(1-p)}$ is the chi-squared distribution with 1 degree-of-freedom.

4. From the definition of the constant $c_{0.95}$,

$$0.95 = \Pr\left[\frac{n\left(\bar{X}-p\right)^2}{p(1-p)} \leq c_{0.95}\right] = \Pr\left[\left(1+\frac{c_{0.95}}{n}\right)p^2 - 2\left(\bar{X}+\frac{c_{0.95}}{2n}\right)p + \bar{X}^2 \leq 0\right].$$

Since

$$\left(\bar{X} + \frac{c_{0.95}}{2n}\right)^2 - \left(1 + \frac{c_{0.95}}{n}\right)\bar{X}^2 = \frac{c_{0.95}\bar{X}(1-\bar{X})}{n} + \left(\frac{c_{0.95}}{2n}\right)^2 > 0,$$

the quadratic equation always has two distinct real solutions. Therefore,

$$\Pr\left[\frac{\bar{X} + \frac{c_{0.95}}{2n}}{1 + \frac{c_{0.95}}{n}} - \frac{\sqrt{\frac{c_{0.95}\bar{X}(1-\bar{X})}{n} + \left(\frac{c_{0.95}}{2n}\right)^2}}{1 + \frac{c_{0.95}}{n}} \le p \le \frac{\bar{X} + \frac{c_{0.95}}{2n}}{1 + \frac{c_{0.95}}{n}} + \frac{\sqrt{\frac{c_{0.95}\bar{X}(1-\bar{X})}{n} + \left(\frac{c_{0.95}}{2n}\right)^2}}{1 + \frac{c_{0.95}}{n}}\right]$$

Using the constant $c_{0.95}$, find the 95% confidence interval of the parameter p.

5. Note that

$$\frac{\bar{X} + \frac{c_{0.95}}{2n}}{1 + \frac{c_{0.95}}{n}} = \frac{\sum_{i=1}^{n} X_i + c_{0.95}/2}{n + c_{0.95}}.$$

In the case of the 95% confidence interval, $c_{0.95} \approx 4$ (When the 97.5% point of the standard normal is given to be 1.96 (≈ 2), the 95% point of the chi-squared distribution with 1 degree-of-freedom is $c_{0.95} = (1.96)^2 \approx 2^2$). Using this fact, the "four plus rule" estimate formula is derived as follows,

$$\frac{\bar{X} + \frac{c_{0.95}}{2n}}{1 + \frac{c_{0.95}}{n}} \approx \frac{\sum_{i=1}^{n} X_i + 2}{n + 4}.$$

This estimation formula is used to avoid unstable estimation of the sample proportion when the estimate is very close to 0 or 1 or when the sample size is relatively small.

経営学

出題の趣旨・解答例

問題 I

本設問の趣旨は，経営戦略論分野における基本知識の習得度を確認することである。ＳＣＰモデルとは，産業組織論にもとづく理論的フレームワークであり，業界構造（structure）によって企業行動（conduct）とパフォーマンス(performance)を説明するモデルである。業界構造はその業界の競合企業の数や製品の均質度，参入と退出のコストなどによって測定される。業界構造の特性は企業が取りうる行動の選択肢に影響を及ぼすが，その状況下で企業は競争優位を築くために価格調整による環境適応や製品差別化，談合などの戦略を追求する。パフォーマンスは企業レベルと社会レベル（たとえば生産と配分の効率性など）という 2 種類に分けられる。SCP モデルの意義として，外部環境の機会や脅威についての分析や事業領域の選択に際して有用であること，などが挙げられる。他方，その限界として，同一業界内の企業間の差異が看過されていること，業界構造と企業行動が動態的に相互に影響を及ぼし合う視点が欠如していることなどが挙げられる。本設問への答案により，（1）経営学，特に経営戦略論の基本概念に関する理解の正確性，(2)修士課程での学修に求められる論理的思考の力量を評価することができる。

Question I.

The aim of this question is to evaluate applicants' understanding of the basic knowledge in the field of strategic management. The S-C-P (structure-conduct-performance) model is a theoretical framework to understand the relationship among industry structure, corporate behavior, and performance. Industry structure is measured by some factors such as the number of competing firms in an industry, homogeneity of products in an industry, cost of entry and exit in an industry. Under the constraint that characteristics of the industry structure define the range of options that firms can take, they pursue the strategies such as price taking, product differentiation, and collusion in order to achieve competitive advantage. Performance can be divided into two types: (1) firm level performance and (2) society level performance such as productive and allocative efficiency. This theoretical model is useful in analyzing environmental opportunities and threats and in considering the options of business areas. However, this model is insufficient to explain the reason why strategies or behavior among firms in a same industry are different, and to understand the dynamic interaction between industrial structure and corporate behavior. The way applicants answer this question reveals both their understanding of the basic concept in business strategy, and their qualifications of logical and reasonable thinking skill.

問題Ⅱ

本設問は，経営組織論における機能別組織構造に関する知識の習得度を確認することを目的としている．解答に際し，①基本的特徴については，組織の基本機能（職能）別に活動がまとめられ部門が設けられている組織であること，②長所としては，各機能内において規模の経済性を高められることや，専門の知識・技能を深めることができる点，③短所としては，部門間のコンフリクトが起こりやすく，経営層に負担がかかるため，環境変化への対応の遅れや，イノベーションが生まれにくいことに触れつつ，適切に説明することが求められる．本設問への解答により，経営組織論の基本概念に関する理解の正確性，および修士課程での学修に求められる論理的な思考力を評価できる．

Question II.

This question aims to assess whether applicants have the basic knowledge of the functional structure in organization theory. Applicants need to explain the following three points. First, the basic characteristics are that departments are grouped by similar functions or work processes. Second, the strengths are that the functional structure allows economies of scale within functional departments and enables in-depth knowledge and skill development. Third, the weaknesses are that the functional structure leads to conflicts among departments, may cause decisions to pile on top managers, slow response time to environmental changes, and results in less innovation.

The way applicants answer this question reveals both their understanding of the key concept in organization theory and their qualifications of logical and reasonable thinking skill.

会計学
出題の趣旨

問題Ⅰ．株式交付費や創立費などの繰延資産が，「過去の取引又は事象の結果として報告主体が支配している経済的資源」といった資産の定義に照らしてどのように位置づけられるかについての思考力を問うている．

問題Ⅱ． 本問は，戦略的コスト・マネジメントとしての原価企画（目標原価計算）について，標準原価計算と対比させながらその特徴を説明することを問うている．特に，以下の点に言及してもらいたい．

・製造段階でのコスト・コントロールと上流管理による原価低減
・フィードバック・コントロールとフィードフォワード・コントロール
・プロダクト・アウトとマーケット・イン
・プッシュ生産方式とプル生産方式

Question I. An asset, which is defined by the IASB, is a present economic resource controlled by the entity as a result of past events. To answer this question reveals that applicants have a skill in thinking about anything from the <u>Conceptual Framework</u>.

Question II. This question asks the respondent to explain the characteristics of target costing as strategic cost management, contrasting it with standard costing. The following characteristics can be pointed out.

・cost control at the production stage and cost reduction through upstream management
・feedback control and feedforward control
・product out and market in
・push production and pull production

オペレーションズ・リサーチ

解説

問題 I.

1．（1）

$$\text{目的関数} \quad \min_{w_0, w_A, w_B} \sigma_A^2 w_A^2 + 2\sigma_{AB} w_A w_B + \sigma_B^2 w_B^2$$

$$\text{制約式} \quad \begin{cases} r w_0 + \mu_A w_A + \mu_B w_B = \mu \\ w_0 + w_A + w_B = 1 \end{cases}$$

（2）最適化問題を解くと，$w_A = 4/13, w_B = 6/13, w_0 = 3/13$.

2．（1）株価×割引率=95 円

（2）取引手数料および貸株のレンタル料はかからないとする．

* 時刻 0 で，現物 A 株式 1 単位を 96 円で売り，同時に
* 現金 95 円で割引国債を買う; そして
* 1 年後に A 株式 1 株を 100 円で買う先渡し契約を結ぶ．
* 1 年後の株式代金は，割引債の償還金を充てる．
* これにより，時刻 0 では手元に 1 円が残る．

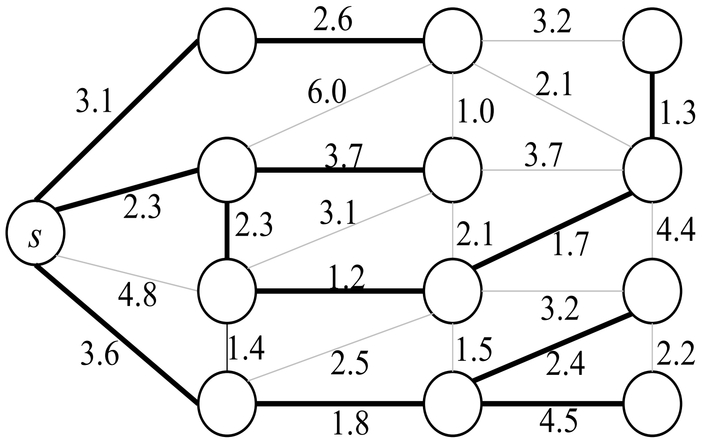
問題 II.

1．(IP) の目的関数と制約式における x_i の係数をそれぞれ a_i と b_i で表す $(i = 1, 2, \ldots, 8)$. 問題 (P) については，$\frac{a_i}{b_i}$ の降順に x_i を見て，可能な限り大きい値を割り当てることで得られる解が最適である．すなわち，$(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (0, 0, 1, 1, 0, 1, 0, \frac{1}{2})$ は最適解で，最適値は，$\frac{495}{2}$ である．

2．$(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (1, 0, 1, 0, 1, 0, 0, 0)$ の目的関数値は 113 である．(IP) の最適値を OPT で表す．1．より，$OPT \leq \frac{495}{2}$ である．$\frac{113}{OPT} \geq \frac{113}{\frac{495}{2}} = \frac{226}{495}$ より，$(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (1, 0, 1, 0, 1, 0, 0, 0)$ の近似比は $\frac{226}{495}$ 以上であるといえる．

問題 III.

下図における太線で示された辺からなる木は，s からの最短路木である．



English Version

Question I.

1.（1）

$$\text{objective function} \quad \min_{w_0,w_A,w_B} \sigma_A^2 w_A^2 + 2\sigma_{AB} w_A w_B + \sigma_B^2 w_B^2$$

$$\text{constraint} \quad \begin{cases} r w_0 + \mu_A w_A + \mu_B w_B = \mu \\ w_0 + w_A + w_B = 1 \end{cases}$$

（2）Solving the optimization problem gives $w_A = 4/13, w_B = 6/13, w_0 = 3/13$.

2.（1）Stock price multiplied by discount rate = 95 yen

（2）Assume no transaction fees and there is no rental fee for stock lending.

* At time 0, sell 1 units of A-share for 96 yen by spot-trading, and at the same time
* buy a discount government bond with 95 cash; and
* Enter a forward contract to buy 1 shares of A-share for 100 yen one year later.

* The redemption amount of the discount bond will be used for the stock payment after one year.
* This leaves 1 yen at time 0.

Question II.

1. For $i = 1, 2, \ldots, 8$, let a_i (resp., b_i) denote the coefficient of x_i in the objective function (resp., the first constraint) for (IP). A solution obtained by sorting x_i in a nonincreasing order of $\frac{a_i}{b_i}$, and taking x_i in this order to assign the highest possible value to it is optimal to (P). Hence, $(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (0, 0, 1, 1, 0, 1, 0, \frac{1}{2})$ is an optimal solution to (P) and the optimal value for (P) is $\frac{495}{2}$.

2. The objective function value for $(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (1, 0, 1, 0, 1, 0, 0, 0)$ is 113. Denote the optimal value for (IP) by OPT. Note that $OPT \leq \frac{495}{2}$ holds by Question 1. Since $\frac{113}{OPT} \geq \frac{113}{\frac{495}{2}} = \frac{226}{495}$ holds, it follows that the approximation ratio of the solution is at least $\frac{226}{495}$.

Question III.

In the following figure, the tree whose edges are drawn in bold is a shortest path tree from s.